

MATH 249: Introduction to Combinatorics (Advanced)

Fall 2022 Course Outline

Instructor: Stephen Melczer

TAs: TBD

Class time: Mondays, Wednesdays, and Fridays from 1:30 – 2:20 in **RCH 305**

Course notes: Distributed on LEARN (you don't need to buy any materials)

This course is scheduled to run **in-person only** (with modifications to be made if necessary to account for any changing health situations).

Overview.

Math 249 is an introduction to combinatorics at an advanced level. This material is closely related to many of the staple puzzle book questions you may have pondered as a kid, and is also foundational to every discipline involving discrete structures. Combinatorics lies in the intersection of pure mathematics, applied mathematics, and computer science, and this course will showcase the intriguing interactions between these areas. The course is primarily theoretical and you will develop your proof-writing skills while working with interesting and useful structures.

The first portion of the course is combinatorial enumeration. If you enjoyed puzzle book questions like “how many ways can you tile a $2 \times n$ board with dominoes?” or ever wondered how the number of binary trees grows, and how to prove it, then you will like this part. Practically, you will learn about generating series, which are one of the most important tools in algebraic enumeration, and see a large selection of combinatorial objects. The second portion of the course is graph theory. If you enjoyed puzzle book questions of the form “can you draw a given figure without lifting your pencil or retracing any line?” or like to think about the structures of networks then you will like this part. Practically you will learn the formal foundations of graph theory, standard results and how to prove them, and some key graph algorithms.

Because Math 249 is an advanced section, we will see all the material covered in Math 239 plus additional topics that go beyond the standard material in interesting and fun ways. These additional topics will be tailored to the interests of students in the class.

Assessments.

The grade breakdown will be

- 30% ten assignments (lowest dropped)
- 35% enumeration exam (midterm)
- 35% graph theory exam (final)

To pass the course you must pass both the test portion and the assignment portion of the assessments.

Exams. The enumeration exam will be written in-person on **Tuesday November 1 from 4:30 – 6:20 in E2 1732 and E2 1736**. The graph theory exam will be scheduled by the registrar and take place in-person during the final exam period (date/time/location will be announced during the term).

Assignments. We will be using the **Crowdmark** system for submitting assignments online. You will receive a Crowdmark link for each assignment, and you must submit your solutions on Crowdmark. You must submit each question in the corresponding box, or it will not be graded. Do not hand in your assignment in person, or it will not be graded. You may write your solutions by hand and upload pictures, or type up your solutions using Latex (remember to separate each question when you upload your answers). **You are strongly encouraged to submit your assignments on Crowdmark well before their deadlines.**

Late assignments can be submitted up to 12 hours after the deadline with a 50% penalty subtracted from your mark on that assignment.

Assignments are to be done individually. You may discuss the assignments in small groups, however **you must write up the solutions on your own.** This means that you may not write up your solutions while you are with a group, and you should not consult any notes you have taken during your group discussions while writing up your solutions. You must **explicitly** acknowledge any discussions with other students by listing their names at the start of your solution (write this for every problem discussed with others).

Many assignment questions are proof questions and you will be evaluated on the logic and presentation of your ideas. Aim to present your proofs at a level that would be understood by an average student in the class who has not thought about the problem yet. You may not use electronic resources for help with assignment questions directly. For example, you may read internet materials to review the definition of a generating function, however, you may not directly search for an assignment question on this (or any other) topic and should immediately stop reading anything you think will solve one of the assignment questions. You are not allowed to use solutions obtained from previous offerings of this course. Any submitted assignments that are suspected of cheating will be reported to the integrity officer of the Faculty of Mathematics.

Piazza. Students are strongly encouraged to help each other understand course materials and content using the forum at piazza.com. We will be using Piazza as a discussion board, and all mathematical questions about course content should be posted there. Guidelines for usage and details about access will be provided early in the term.

Schedule and material.

Course resources and announcements will be posted on LEARN. Here is a **tentative** schedule with topics that we plan to cover. The breakdown of topics between weeks may change as we go through the course.

| Week | Start | Topics (Tentitive) | Assignments (Thursdays) |
|---|---------|--------------------------------------|-------------------------|
| <i>Enumeration Portion</i> | | | |
| Week 1 | Sept 7 | bijections and combinatorial proofs | A1 Released |
| Week 2 | Sept 12 | generating functions | A1 Due and A2 Released |
| Week 3 | Sept 19 | combinatorial classes | A2 Due and A3 Released |
| Week 4 | Sept 26 | applications of generating functions | A3 Due and A4 Released |
| Week 5 | Oct 3 | recurrences | A4 Due and A5 Released |
| <i>Reading Week October 10 – 15</i> | | | |
| Week 6 | Oct 17 | additional material | A5 Due and A6 Released |
| Week 7 | Oct 24 | additional material | A6 Due |
| Enumeration Exam Tuesday Nov 1 (4:30–6:20) | | | |
| <i>Graph Theory Portion</i> | | | |
| Week 8 | Oct 31 | graphs and connectedness | A7 Released |
| Week 9 | Nov 7 | walks and spanning trees | A7 Due and A8 Released |
| Week 10 | Nov 14 | planarity | A8 Due and A9 Released |
| Week 11 | Nov 21 | graph colouring and matchings | A9 Due and A10 Released |
| Week 12 | Nov 28 | additional material | A10 Due |
| Week 13 | Dec 5 | final class (course summary) | |
| Graph Theory Exam TBD | | | |

First class is **Wednesday** September 7. Final class is **Monday** December 5. Assignments are due by 10pm.

Computer Software: I will occasionally use the (free and open source) computer algebra software Sage to illustrate some examples during lectures. Although it is not necessary to learn Sage for the course, I will release the code used to generate these examples and playing around with it yourself can greatly help your understanding. See <https://melczer.ca/249/> or LEARN for information on installing Sage and a basic tutorial on how to use it.

Administrative policy

Academic integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. **You must do individual work on your own, and you must not do other people's individual work or seek out other people to do yours. You must not upload course material to sharing sites such as chegg nor seek out information there.** Check the [Office of Academic Integrity](#) for more information.

Grievance: A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read [Policy 70, Student Petitions and Grievances, Section 4](#). When in doubt, please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity to avoid committing an academic offence, and to take responsibility for his/her actions (check the [Office of Academic Integrity](#) for more information). A student who is unsure whether an action constitutes an offence, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course instructor, academic advisor, or the undergraduate associate dean. For information on categories of offenses and types of penalties, students should refer to [Policy 71, Student Discipline](#). For typical penalties, check [Guidelines for the Assessment of Penalties](#).

Appeals: A decision made or penalty imposed under [Policy 70, Student Petitions and Grievances](#) (other than a petition) or [Policy 71, Student Discipline](#) may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to [Policy 72, Student Appeals](#).

Note for students with disabilities: [AccessAbility Services](#), located in Needles Hall, Room 1401, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with AccessAbility Services at the beginning of each academic term.

Mental Health Support: I encourage you to seek out mental health support if needed. On-campus Resources include

- Campus Wellness: <https://uwaterloo.ca/campus-wellness/>
- Counselling Services: <https://uwaterloo.ca/campus-wellness/counselling-services> (519-888-4567 ext 32655)
- MATES: one-to-one peer support program offered by Federation of Students (FEDS) and Counselling Services (email mates@uwaterloo.ca)
- Health Services: located across the creek from the Student Life Centre (519-888-4096)

Off-campus Resources include

- Good2Talk (24/7): Free confidential help line for post-secondary students (phone 1-866-925-5454)
- Here 24/7: Mental Health and Crisis Service Team (phone 1-844-437-3247)
- OK2BME: set of support services for lesbian, gay, bisexual, transgender or questioning teens in Waterloo (phone 519-884-0000 extension 213)

Diversity: It is my intent that students from all backgrounds and perspectives be well served by this course, and that students' learning needs be addressed both in and out of class. I recognize the immense value of the diversity in identities, perspectives, and contributions that students bring, and the benefit it has on our educational environment. Your suggestions are encouraged and appreciated. Please let me know ways to improve the effectiveness of the course for you personally or for other students or student groups. In particular:

- I will gladly honour your request to address you by an alternate/preferred name or gender pronoun. Please advise me of this preference early in the semester so I may make appropriate changes to my records.
- I will honour your religious holidays and celebrations. Please inform of me these at the start of the course.
- I will follow AccessAbility Services guidelines and protocols on how to best support students with different learning needs.